Energy Evolution for the Sivers Asymmetries in Hard Processes

Peng Sun[1] and Feng Yuan[1]

[1]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*

Abstract

We investigate the energy evolution of the azimuthal spin asymmetries in semi-inclusive hadron production in deep inelastic scattering (SIDIS) and Drell-Yan lepton pair production in pp collisions. The scale dependence is evaluated by applying an approximate solution to the Collins-Soper-Sterman (CSS) evolution equation at one-loop order which is adequate for moderate Q^2 variations. This describes well the unpolarized cross sections for SIDIS and Drell-Yan process in the Q^2 range of 2.4-100GeV^2. A combined analysis of the Sivers asymmetries in SIDIS from HERMES and COMPASS experiments, and the predictions for the Drell-Yan process at RHIC at $\sqrt{S} = 200$GeV are presented. We further extend to the Collins asymmetries and find, for the first time, a consistent description for HERMES/COMPASS and BELLE experiments with the evolution effects. We emphasize an important test of the evolution effects by studying di-hadron azimuthal asymmetry in e^+e^- annihilation at moderate energy range, such as at BEPC at $\sqrt{S} = 4.6$GeV.

Introduction. Transverse spin azimuthal angular asymmetries in hadronic processes have attracted great attentions in recent years. This is not only because the associated observables are keen to provide important information on nontrivial hadronic structures, but also because they are sensitive to the strong interaction dynamics [1]. The latter involves core feature of quantum chromodynamics (QCD): the factorization and universality of the associated parton distributions and fragmentation functions, and the energy evolution in hard scattering processes.

Among of these observables the major focuses are the Sivers and Collins asymmetries in semi-inclusive hadron production in deep inelastic scattering (SIDIS) and Drell-Yan lepton pair production in pp collisions, and di-hadron production in e^+e^- annihilation processes. The Sivers effects come from the asymmetric transverse momentum dependent (TMD) parton distribution in nucleon which correlates with the transverse polarization vector $S_\perp$, whereas the Collins effects come from the similar correlation in the fragmentation process associated with the quark polarization. However, these two functions have different universality properties: Sivers function differs by sign between SIDIS and Drell-Yan processes [2, 3], while the Collins function is universal between SIDIS and e^+e^- processes. Both asymmetries have been observed in SIDIS from HERMES, COMPASS, and JLab Hall A experiments [4–7]. In addition, Collins asymmetry has been observed in e^+e^- process by BELLE collaboration [8].

The experimental test of the above universality, in particular, for the Sivers asymmetries between Drell-Yan and SIDIS, is one of top questions in hadronic physics. However, the Sivers asymmetries were observed in SIDIS with Q^2 around 3GeV^2, whereas the Drell-Yan processes will be measured in the range that is greater than 20GeV^2. In order to consolidate the universality test, the Q^2 dependence of the Sivers asymmetry must be understood correctly. The theoretical framework to study the energy evolution of these observables has been well developed, where the Collins-Soper-Sterman (CSS) equation [9, 10] for both spin-average and single-spin dependent cross section has been derived [11–16]. The CSS formalism has been applied successfully to describe low transverse momentum distribution of vector boson (Drell-Yan, W/Z) production in unpolarized pp collisions (see, for example, Ref. [17]). Early estimate for Q^2 dependence of the SSA [11] was limited to high Q^2 range. A recent calculation found a surprising strong evolution effects from HERMES/COMPASS energies to typical Drell-Yan energy [18]. This evolution formalism was later applied in a fit to HERMES/COMPASS data [19]. The result of Ref. [18] has raised great concerns in the experimental proposals, since the predicted asymmetries for Drell-Yan processes would be too small due to the evolution. In this paper, we will examine these studies, and carefully investigate the Q^2 evolution of both spin average and single-spin dependent cross sections. By doing so, we find that the previous study of Ref. [18] over-estimated the evolution effects. In particular, the transverse momentum spectrum of the Drell-Yan process in the relevant Q^2 range can not be described by the TMD quark distributions proposed in Ref. [15, 18] (see Fig. 1 below).

In our calculation, we take an alternative approach, following the original suggestion of Ref. [12, 13], by directly applying the CSS equation at one-loop order from low to high energies. The one-loop evolution kernel contains a term which predicts a P_T broadening effects at higher Q^2. We will show that this can describe the transverse momentum distribution for both SIDIS and Drell-Yan processes, which cover Q^2 in the range of 2.4-100GeV^2. We extend the evolution to the Sivers asymmetries in these processes, and perform a combined fit to the HERMES and COMPASS data. The predictions for the SSA in Drell-Yan process at RHIC

will be updated with the evolution effects. Finally, we will apply the evolution equation to the Collins asymmetries in SIDIS and di-hadron production in e^+e^- annihilation.

Collins-Soper-Sterman Evolution. We take the SIDIS as an example, where $e(\ell)+p(P) \to e(\ell')+h(P_h)+X$, which proceeds through exchange of a virtual photon with momentum $q_\mu = \ell_\mu - \ell'_\mu$ and invariant mass $Q^2 = -q^2$. When $P_{h\perp} \ll Q$, the transverse-momentum-dependent factorization formalism applies, according which the differential SIDIS cross section can be written as

$$\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2\vec{P}_{h\perp}} = \sigma_0 \times \left[F_{UU} + \epsilon^{\alpha\beta} S_\perp^\alpha F_{\rm sivers}^\beta\right] , \tag{1}$$

where $\sigma_0 = 4\pi\alpha_{\rm em}^2 S_{ep}/Q^4 \times (1-y+y^2/2)x_B$, and y, x_B, and z_h are usual kinematics for SIDIS. We only keep the terms we are interested in: F_{UU} corresponds to the unpolarized cross section, and $F_{\rm sivers}$ to the Sivers function contribution to the single-transverse-spin asymmetry. F_{UU} and $F_{\rm sivers}$ depend on the kinematical variables, x_B, z_h, Q^2, and $P_{h\perp}$, can be written into a factorization form with TMD quark distribution and fragmentation functions and soft and hard factors. The Q^2 dependence of $F_{UU,\rm sivers}$ can be calculated from perturbative QCD, and is controlled by the CSS evolution equation, which is easily formulated in the impact parameter space. For example, for $F_{\rm sivers}^\alpha(Q; P_{h\perp}) = \int \frac{d^2b}{(2\pi)^2} e^{i\vec{P}_{h\perp}\cdot\vec{b}/z_h} \widetilde{F}_{\rm sivers}^\alpha(Q; b)$ [1], we have [10],

$$\widetilde{F}_{\rm sivers}^\alpha(Q; b) = \widetilde{F}_{\rm sivers}^\alpha(Q_0; b) e^{-\mathcal{S}_{Sud}(Q,Q_0,b)} . \tag{2}$$

The perturbative calculable evolution effect has been included in the Sudakov form factor $\mathcal{S}_{Sud}$. In the complete CSS resummation, Q_0 was set at $1/b$, and the b_* prescription was introduced: $b_* = b/\sqrt{1+b^2/b_{max}^2}$ to deal with the Landau pole singularity [10]. This necessarily introduces a non-perturbative form factor [10], which can be determined by comparison to the experimental data [17].

Alternatively, it was argued in Refs. [12, 13] that we can avoid the Landau pole singularity by a direct integration from low to high energy scale,

$$\mathcal{S}_{Sud} = 2C_F \int_{Q_0}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2}\right] , \tag{3}$$

where $c_0 = 2e^{-\gamma_E}$ with the complete one-loop coefficients from a recent calculation [20], and both Q and Q_0 are chosen in the perturbative region. Because of the residual log term in the integral, the above Sudakov form factor is not the complete solution to the CSS evolution. But, it is a reasonable approximation in the moderate Q and Q_0 range, in particular, between the HERMES/COMPASS and typical Drell-Yan energy regions. For very large Q, such as W/Z boson production in pp collisions, we have to take into account higher order corrections and back to the CSS formalism. We notice that the same Sudakov form factor applies to both $\widetilde{F}_{UU}$ and $\widetilde{F}_{\rm sivers}$ since they share the same evolution kernel and hard factors in the TMD factorization. It works for the Drell-Yan lepton pair production in pp collisions as well [2].

Before we study the energy evolution of the SSA, we shall check the above equation can describe the spin-average cross sections in the relevant energy range. The majority of the SIDIS data from HERMES and COMPASS are in a relative low Q^2 range. Therefore,

[1] Here we only keep the dominant term at low $P_{h\perp}$ region, and neglect higher power correction of $P_{h\perp}/Q$.
[2] The difference in hard factors in the TMD factorization does not affect the evolution equation.

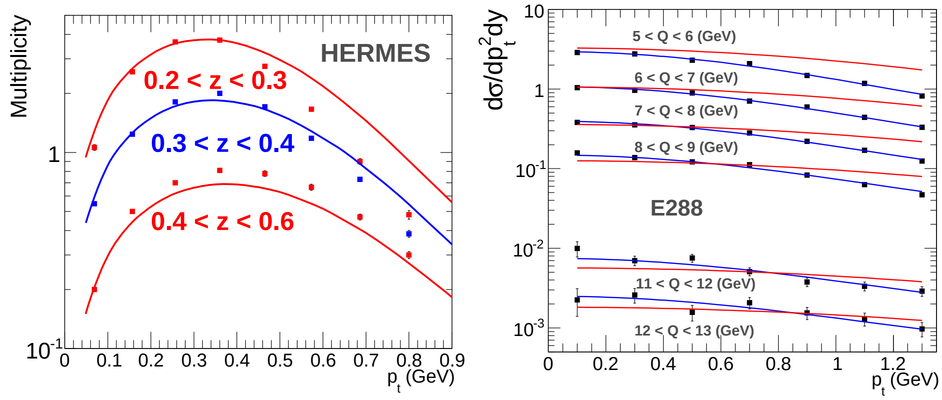

FIG. 1: Comparison between the theory predictions with experimental data of the low transverse momentum distribution of SIDIS at $Q^2 = 2.4\text{GeV}^2$ Ref. [24] and Drell-Yan lepton pair production in pp collisions with various Q^2 range Ref. [25]. The scale evolution comes from the Sudakov form factor of Eq. (3). The predictions calculated from the TMD quark distributions of Ref. [15] with $b_{max} = 0.5GeV^{-1}$ and $g_2 = 0.65GeV^2$ are also shown as red curves in the right panel.

we set the lower scale Q_0 around these experiments, where a Gaussian assumption for the TMD quark distribution and fragmentation functions can well describe the data [21, 22]. Translating this into the impact parameter space, we parameterize $\widetilde{F}_{UU}$ as

$$\widetilde{F}_{UU}(Q_0, b) = \sum_q e_q^2 \, f_q(x_B) \, D_q(z_h) e^{-g_0 b^2 - g_h b^2/z_h^2} , \tag{4}$$

at $Q_0^2 = 2.4GeV^2$, where $f_q(x_B)$ and $D_q(z_h)$ represent the quark distribution and fragmentation functions following the CTEQ and DSS set Ref [23] at lower scale $Q_0^2 = 2.4GeV^2$. In the above equation, g_0 and g_h represent the transverse momentum dependence coming from the distribution and fragmentation, respectively. In the left panel of Fig. 1, we compare the above prediction to the multiplicity distribution in SIDIS from HERMES experiment [24] , where we have chosen $g_0 = 0.097$ and $g_h = 0.045$. These parameters agree well with those in Ref. [18, 22].

We can study the Q^2 evolution by comparing to the fixed target Drell-Yan process, with Q^2 range from 20 to 100 GeV^2. To calculate the transverse momentum spectrum for this process, we apply the universality of the TMD quark distributions, and the evolution equation from Q_0 scale to higher Q. We plot the comparisons between the theory calculations with the experimental data in the right panel of Fig. 1. The broadening effects for the Drell-Yan processes are well reproduced by the evolution effects of Eqs. (2,3). For comparison, we also plot the predictions from the TMD quark distributions calculated from Ref. [18] with their evolution effects. Clearly, Ref. [18] over-estimate the broadening effects. It is caused by a modification of the non-perturbative form factors used in Ref. [17] in order to describe the current SIDIS data, which unfortunately breaks the original predictions for the Drell-Yan processes in Ref. [17].

Sivers Asymmetries in SIDIS and Drell-Yan. Now, we turn to the Sivers single spin asymmetries in SIDIS and Drell-Yan processes. Similar to the above, we parameterize $\widetilde{F}^{\alpha}_{\text{sivers}}$

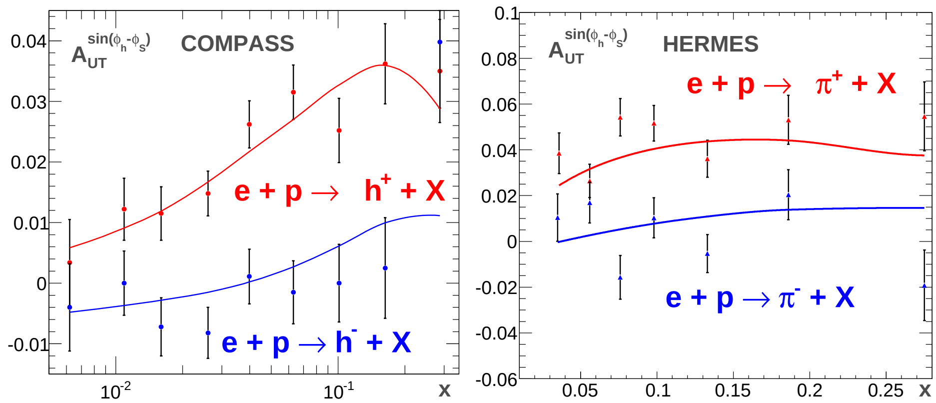

FIG. 2: Theory fit to the experimental data on Sivers single spin asymmetries in SIDIS , as functions of x_B: left panel from COMPASS [7] and right from HERMES [4]. Q^2 evolution has been taken into account from Eq. (3).

at low energy scale $Q_0^2 = 2.4GeV^2$,

$$\widetilde{F}^{\alpha}_{\rm sivers}(Q_0, b) \;=\; \frac{b_\perp^\alpha M}{2} \sum_q \; e_q^2 \; \Delta f_q^{\rm sivers}(x) \; D_q(z) e^{-(g_0-g_s)b^2 - g_h b^2/z_h^2} \;, \tag{5}$$

where $M = 1GeV$ is a normalization scale, and we have chosen an additional parameter g_s for transverse momentum dependence and the fragmentation part remains the same. The function $\Delta f_q(x) = N_q x^{\alpha_q}(1-x)^{\beta_q} \frac{(\alpha_q+\beta_q)^{\alpha_q+\beta_q}}{\alpha_q^{\alpha_q}\ \beta_q^{\beta_q}} f_q(x)$ parameterize the x-dependence of the quark Sivers function similar to that in Ref. [21]. We have the following free parameters: g_s, α_q, β_q and N_q for valence up, down, and sea quarks. Since the data are not sufficient to differentiate g_s for different flavors, we choose the same g_s. We further assume the same sea quark parameterization for up, down and strange quarks.

TABLE I: Parameters $\{a_i^0\}$ describing our optimum Δf_i in Eq. (5) at the input scale $Q^2 = 2.4\,$GeV.

flavor i	N_i	α_i	β_i	g_s (GeV^2)
u	0.1±0.004	0.88±0.088	4.1±0.58	0.024±0.005
d	-0.5±0.010	0.23±0.100	5.3±1.13	0.024±0.005
sea	0.5±0.133	0.43±0.107	0.016±0.17	0.024±0.005

With the above parameterization and the energy evolution effects taken by Eqs. (2,3) for both spin-average and single-spin-dependent cross sections, we perform a combined fit to the Sivers asymmetries from HERMES and COMPASS experiments which scans Q^2 ∼2.4-10GeV^2. We have total of 420 data points, with a minimum χ^2 fit. The best fit results into $\chi^2/d.o.f = 1.67$ and the parameters listed in Table I. As an example, we show in Fig. 2 the comparisons between the theory calculations and the experimental data as functions of x_B for COMPASS and HERMES experiments, which demonstrate a consistent description of both data.

Having constrained quark Sivers functions from HERMES/COMPASS experiments, we will be able to make predictions for the SSAs in the Drell-Yan processes with the evolution

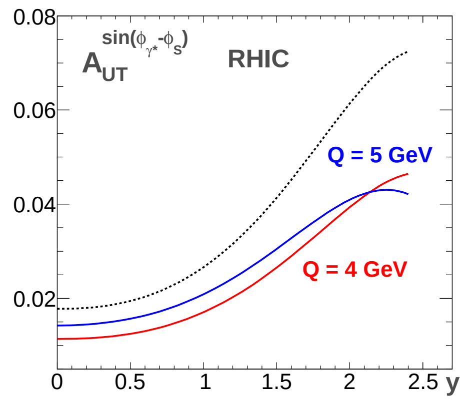

FIG. 3: Predictions for the Sivers single spin asymmetries of Drell-Yan lepton pair production at RHIC, $\sqrt{S} = 200$GeV, as functions of rapidity for two different mass ranges. As a comparison, we also show the prediction without the evolution effects for $Q = 4$GeV case as dotted line.

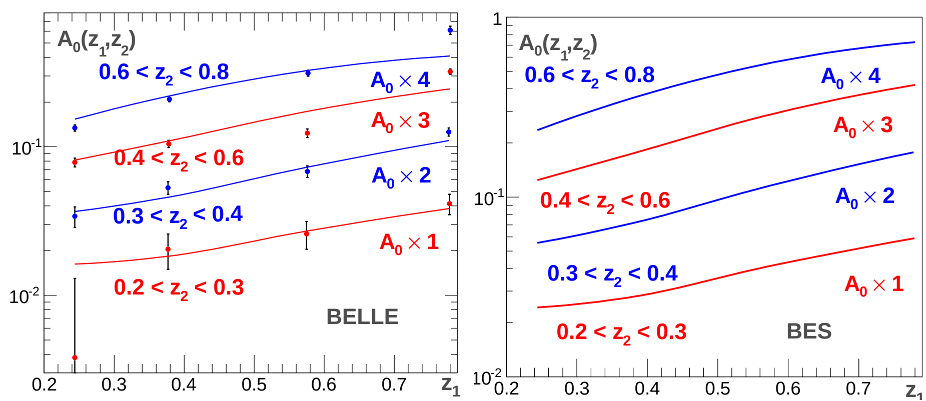

FIG. 4: The Collins asymmetries in di-hadron azimuthal angular distributions in e^+e^- annihilation processes: fit to the BELLE experiment at $\sqrt{S} = 10.6$GeV Ref. [8], and predictions for the experiment at BEPC at $\sqrt{S} = 4.6$GeV.

effects. In Fig. 3, we show that for RHIC experiment at $\sqrt{S} = 200$GeV, as function of rapidity with $P_\perp$ integrated up to 2GeV. We have flipped the sign for the quark Sivers function because of the nontrivial universality property for the Sivers function. For comparison, we have also plotted the prediction without the evolution effects by setting $\mathcal{S}_{Sud} = 0$ in Eq. (3). From this, we see that the evolution reduces the asymmetry by about a factor of 2. This is different from that in Ref [18], where an order of magnitude reduction was indicated for the typical Drell-Yan experiments.

We have done a number of cross checks for the above evolution effects. First, we can tune the parameter in the calculations of Ref. [18] to reproduce the $P_\perp$ spectrum of the Drell-Yan data, which leads to a much smaller $g_2 = 0.09$. With that change, we can describe both SIDIS and Drell-Yan data in Fig. 1, and the predicted SSA would be in the similar range as ours in Fig. 3. Second, we determine the transverse momentum moment of the quark Sivers function (Qiu-Sterman matrix element) from the fit in Fig. 2, and calculate the SSA in Drell-Yan process by using the resummation formula in Ref. [20], neglecting the scale dependence of the integrate parton distributions and correlation functions and assuming the non-perturbative form factor from Ref. [26] for Drell-Yan process with a mild x-dependence.

Again, we obtain the prediction in a similar range as that in Fig. 3. In particular, this method provides an important step to matching the SIDIS to Drell-Yan and W/Z boson productions in pp collisions.

Finally, we turn to the energy evolution of the Collins asymmetries. We perform a combined analysis to the Collins asymmetries in SIDIS from HERMES and COMPASS experiments and the di-hadron azimuthal angular asymmetries in e^+e^- annihilation from BELLE experiment [8]. Again, we parameterize the Collins function at low energy scale $Q_0^2 = 2.4GeV^2$ as $\widetilde{H}_1^{\perp\alpha}(z, b_\perp) = \left(\frac{-ib_\perp^\alpha M}{2z}\right) e^{-(g_h-g_c)b^2/z^2} N_q z^{\alpha_q}(1-z)^{\beta_q} \frac{(\alpha_q+\beta_q)^{\alpha_q+\beta_q}}{\alpha_q^{\alpha_q}\ \beta_q^{\beta_q}} D_q(z)$ and take the evolution effects of Eqs. (3). The overall fit is very good, as we show the comparison between the theory predictions and the BELLE data. The fitting parameters for the Collins functions are listed in Table II, with a $\chi^2/d.o.f = 1.2$.

TABLE II: The fitting parameters $\{a_i^0\}$ for the Collins function at the input scale $Q^2 = 2.4\,\mathrm{GeV}$.

flavor i	N_i	α_i	β_i	g_c (GeV^2)
u	0.34±0.006	3.9±0.60	0.85±0.26	0.012±0.002
d	-0.34±0.011	0.4±0.26	0.26±0.33	0.012±0.002

An important feature we found in this fit is that both favored and disfavored Collins functions saturates the positivity bounds. Therefore, it is very important to check the energy dependence in other experiments. One idea place is the planed electron-ion colliders [1], where SIDIS processes with wide Q^2 coverage are the major focuses in the proposal. Another place is the e^+e^- annihilation process at different energies. We suggest to investigate the di-hadron azimuthal correlation in e^+e^- annihilation at the BEPC of IHEP, Beijing, which can reach to the center of mass energy around $\sqrt{S} = 4.6$GeV. We show the prediction for that energy in right panel of Fig. 3. Earlier experiments at SLAC around the similar energy range demonstrated applicability of perturbative QCD description of the jet structure [27], which shall support to pursue similar studies at BEPC including the Collins asymmetries. The initial state radiation events at BELLE can also be used to study this asymmetry in various energies [28].

Summary. In this paper, we have investigated the energy scale dependence of the spin and azimuthal angular asymmetries in hard scattering processes. We applied the Collins-Soper-Sterman evolution at one-loop order in the moderate energy range, which can well describe the transverse momentum spectrum in existing SIDIS and Drell-Yan data. We focused on the energy dependence of the Sivers and Collins asymmetries in these processes, and performed a combined analysis with the existing experimental data, including HERMES, COMPASS, and BELLE experiments. The non-perturabtive TMD Sivers function and Collins fragmentation functions are determined. The predictions for future experiments are also presented. These experiments will provide an important test for the TMD universality and strong interaction dynamics. We will present more detailed results of this calculation, and the matching from SIDIS to Drell-Yan and W/Z boson production in pp collisions in a separate publication.

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